Nuclear Structure and IBM

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Nuclear structure refers to the arrangement and behaviour of protons and neutrons (collectively known as nucleons) within the atomic nucleus. The study of nuclear structure is fundamental to understanding how atomic nuclei behave, their stability, and the interactions that govern nuclear reactions. The key factors influencing nuclear structure include:

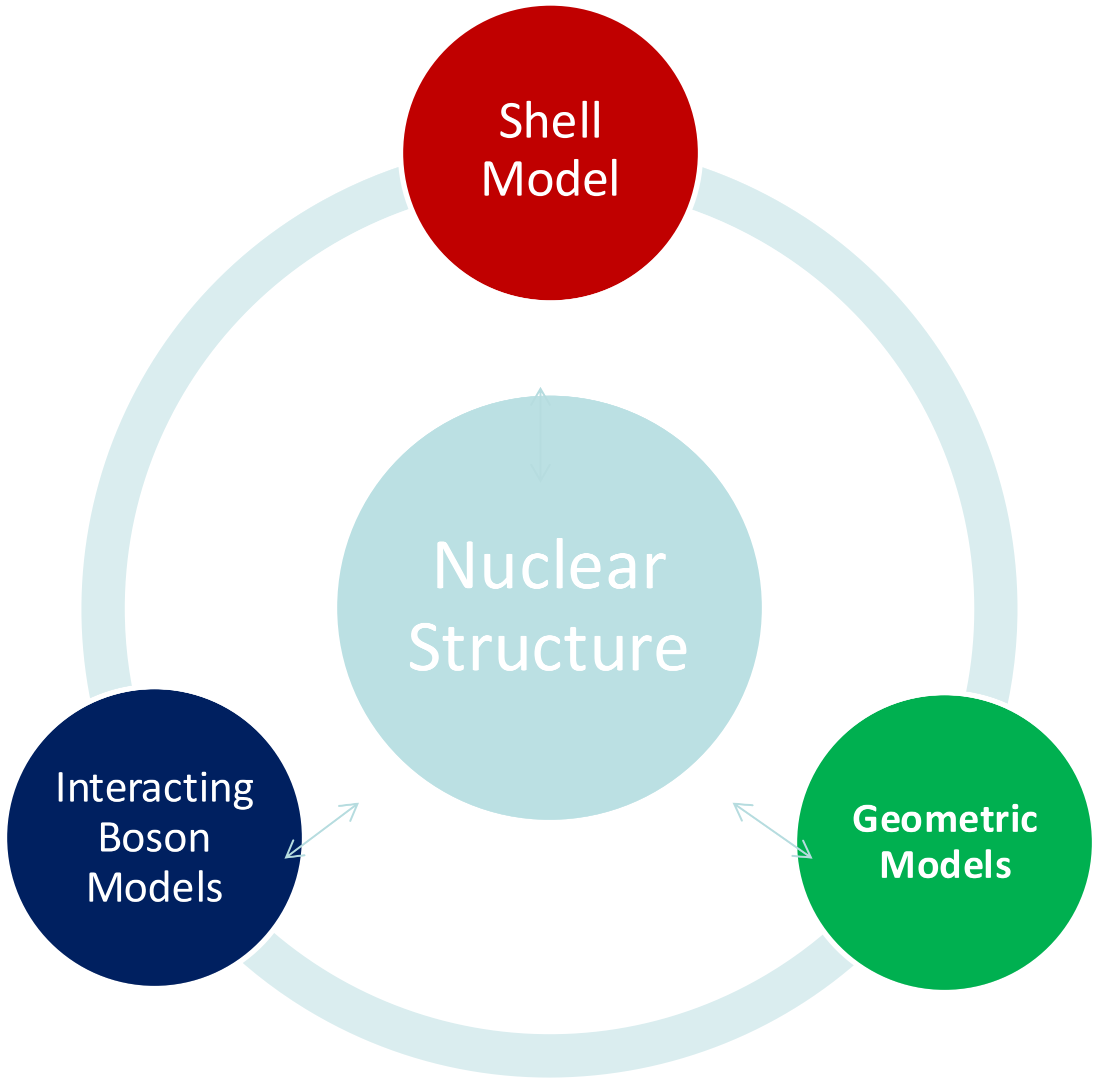
1.Nucleon-Nucleon Interactions: Protons and neutrons interact through the nuclear force, which binds them together. These interactions are complex, involving both short-range attractive forces and long-range repulsive forces.

A. N. Mitra Memorial Lecture, April 14, 15, 2025

2. Shell Model: This is a simple, yet powerful, model for describing the structure of the nucleus. It treats protons and neutrons as moving in discrete energy levels or shells, much like electrons in atomic shells. The shell model helps explain the stability of certain isotopes and the properties of the nucleus, such as spin and parity.

3.Collective Model: This model describes the nucleus as a system that can exhibit collective behavior, like rotations and vibrations, due to the collective motion of nucleons. It accounts for phenomena such as nuclear deformation (spherical, prolate, or oblate shapes) and nuclear excitations.

4.Deformation and Shape: Nuclei may not be spherical; they can take on prolate (elongated) or oblate (flattened) shapes. This deformation is particularly important for heavier nuclei and can lead to unique phenomena such as nuclear rotations.


Shell
Model
Nuclear
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Interacting
Boson
Models
Geometric
Models

5. Pairing and Collective Excitations: In certain nuclei, pairs of protons or neutrons interact in a correlated way, leading to collective excitations. These correlations are a key feature in nuclear structure and can result in behaviors such as superfluidity and the formation of collective vibrational states.

Interacting Boson Model (IBM):

The IBM is a theoretical framework used to describe the collective aspects of nuclear structure, particularly the low-energy excitations in atomic nuclei. It was developed by **F. Iachello** and **A. Arima** in the 1970s. The IBM has become an essential tool for understanding collective motion, such as vibrations and rotations, in even-even nuclei (nuclei with an even number of protons and neutrons).

1.**Key Concepts of the IBM:**

Boson Representation: The IBM treats nucleons (protons and neutrons) as collective excitations that can be represented by bosons. A boson is a particle that obeys Bose-Einstein statistics, and in the IBM, the nucleons are approximated as bosonic pairs. These pairs can be of two types:

S-bosons: These represent pairing of nucleons in the **s-state** (orbital angular momentum L=0).

D-bosons: These represent pairing of nucleons in the **d-state** (orbital angular momentum L=2).

2. Hamiltonian: The dynamics of the IBM are described by an effective Hamiltonian that includes terms representing the interactions between the bosons. These interactions can be modeled by terms such as:
Boson-boson interaction: Describes the attractive forces between pairs of nucleons, leading to collective motion.
Single-particle excitations: Models the transition between different collective states.

3. **Symmetry Groups**: The IBM uses group theoretical methods to classify nuclear states. The two main symmetries in the IBM are:
U(6) Symmetry: This is the fundamental symmetry group of the IBM, describing the full space of bosonic configurations.
SU(3) Symmetry: For certain nuclei, the IBM can reduce to SU(3) symmetry, which describes the rotational behavior of deformed nuclei. This symmetry is often associated with the rotation of a deformed, ellipsoidal shape of the nucleus.

4. Phases of Nuclear Collective Motion:

The IBM describes various phases of nuclear collective motion:

U(5) limit: This corresponds to spherical nuclei and represents a vibrational model where the nucleus undergoes collective vibrational excitations.

SU(3) limit: This corresponds to prolate or oblate deformed nuclei and represents a rotational model where the nucleus undergoes collective rotational motion.

O(6) limit: This corresponds to nuclei with soft shapes, where the transition between vibrational and rotational modes is more flexible.

Interacting boson approximation

Dominant interaction between nucleons has pairing character $\Rightarrow$ two nucleons form a pair with angular momentum *J*=0 (*S* pair).

Next important interaction between nucleons with angular momentum *J*=2 (*D* pair).

Approximation: Replace *S* and *D* fermions pairs by *s* and *d* bosons. Argument:

$$\left[\hat{S},\hat{S}^{+}\right]=1-\frac{\hat{n}}{\Omega}\approx 1 \text{ while } \left[s,s^{+}\right]=1$$

Microscopy of IBM

In a boson mapping, fermion pairs are represented as bosons:

$$s^{+} \Leftrightarrow \hat{S}^{+} \equiv \sum_{j} \alpha_{j} \left(a_{j}^{+} \times a_{j}^{+}\right)_{0}^{(0)}, \quad d_{\mu}^{+} \Leftrightarrow \hat{D}_{\mu}^{+} \equiv \sum_{jj'} \beta_{jj'} \left(a_{j}^{+} \times a_{j'}^{+}\right)_{\mu}^{(2)}$$

Mapping of operators (such as Hamiltonian) should take account of Pauli effects.

Two different methods by

requiring same commutation relations;

associating state vectors.

T. Otsuka *et al.*, Nucl. Phys. A **309** (1978) 1

The interacting boson model

Describe the nucleus as a system of *N* interacting *s* and *d* bosons. Hamiltonian:

$$\hat{H}_{\mathrm{IBM}} = \sum_{i=1}^{6} \varepsilon_i b_i^+ b_i + \sum_{ijkl=1}^{6} \upsilon_{ijkl} b_i^+ b_j^+ b_k b_l$$

Justification from

Shell model (SM): s and d bosons are associated with S and D fermion (Cooper) pairs.

Geometric model (GM): for large boson number the IBM reduces to a liquid-drop Hamiltonian.

A. Arima & F. Iachello, Ann. Phys. (NY) **99** (1976) 253; **111** (1978) 201; **123** (1979) 468

U(6) algebra and symmetry

Introduce 6 creation & annihilation operators:

$$\left\{b_i^+, i=1,\ldots,6\right\} = \left\{s^+, d_{-2}^+, d_{-1}^+, d_0^+, d_{+1}^+, d_{+2}^+\right\}, \quad b_i = \left(b_i^+\right)^+$$

The hamiltonian (and other operators) can be written in terms of generators of U(6):

$$\left[b_i^+ b_j, b_k^+ b_l\right] = b_i^+ b_l \delta_{jk} - b_k^+ b_j \delta_{il}$$

The harmonic hamiltonian has U(6) symmetry

$$\hat{H}_{\mathrm{U(6)}} = E_0 + \sum_{i=1}^{6} b_i^+ b_i \Rightarrow \left[\hat{H}_{\mathrm{U(6)}}, b_i^+ b_j\right] = 0$$

Additional terms break U(6) symmetry.

The IBM Hamiltonian

Rotational invariant Hamiltonian with up to *N*-body interactions (usually up to 2):

$$\hat{H}_{\mathrm{IBM}} = E_0 + \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d + \sum_{l_1 l_2 l_1' l_2', L} \upsilon^L_{l_1 l_2 l_1' l_2'} \left(b^+_{l_1} \times b^+_{l_2}\right)^{(L)} \cdot \left(\tilde{b}_{l_1'} \times \tilde{b}_{l_2'}\right)^{(L)}$$

The single-boson energies ε and boson-boson interactions υ is the IBM Hamiltonian are solvable.

This problem is equivalent to the enumeration of all algebras *G* satisfying

$$\mathrm{U}(6) \supset G \supset \mathrm{SO}(3) \equiv \left\{ \hat{L}_\mu = \sqrt{10}\left(d^+ \times \tilde{d}\right)^{(1)}_\mu \right\}$$

Dynamical symmetries of the IBM

U(6) has the following subalgebras:

$$\mathrm{U}(5)=\left\{\left(d^+\times\tilde{d}\right)^{(0)}_\mu,\left(d^+\times\tilde{d}\right)^{(1)}_\mu,\left(d^+\times\tilde{d}\right)^{(2)}_\mu,\left(d^+\times\tilde{d}\right)^{(3)}_\mu,\left(d^+\times\tilde{d}\right)^{(4)}_\mu\right\}$$

$$\mathrm{SU}(3)=\left\{\left(d^+\times\tilde{d}\right)^{(1)}_\mu,\left(s^+\times\tilde{d}+d^+\times\tilde{s}\right)^{(2)}_\mu-\sqrt{\tfrac{7}{4}}\left(d^+\times\tilde{d}\right)^{(2)}_\mu\right\}$$

$$\mathrm{SO}(6)=\left\{\left(d^+\times\tilde{d}\right)^{(1)}_\mu,\left(s^+\times\tilde{d}+d^+\times\tilde{s}\right)^{(2)}_\mu,\left(d^+\times\tilde{d}\right)^{(3)}_\mu\right\}$$

$$\mathrm{SO}(5)=\left\{\left(d^+\times\tilde{d}\right)^{(1)}_\mu,\left(d^+\times\tilde{d}\right)^{(3)}_\mu\right\}$$

Three solvable limits are found:

$$\mathrm{U}(6)\supset\left\{\begin{array}{c}\mathrm{U}(5)\supset\mathrm{SO}(5)\\ \mathrm{SU}(3)\\ \mathrm{SO}(6)\supset\mathrm{SO}(5)\end{array}\right\}\supset\mathrm{SO}(3)$$

Dynamical symmetries of the IBM

The general IBM Hamiltonian is

$$\hat{H}_{\mathrm{IBM}} = E_0 + \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d + \sum_{l_1 l_2 l_1' l_2', L} \upsilon^{L}_{l_1 l_2 l_1' l_2'} \left(b^+_{l_1} \times b^+_{l_2}\right)^{(L)} \cdot \left(\tilde{b}_{l_1'} \times \tilde{b}_{l_2'}\right)^{(L)}$$

An *entirely equivalent* form of H_{IBM} is

$$\begin{aligned}\hat{H}_{\mathrm{IBM}} = E_0 &+ \eta_0 \hat{C}_1[\mathrm{U}(6)] + \eta_1 \hat{C}_1[\mathrm{U}(5)] + \kappa_0' \hat{C}_1[\mathrm{U}(6)]\hat{C}_1[\mathrm{U}(5)] \\ &+ \kappa_0 \hat{C}_2[\mathrm{U}(6)] + \kappa_1 \hat{C}_2[\mathrm{U}(5)] + \kappa_2 \hat{C}_2[\mathrm{SU}(3)] \\ &+ \kappa_3 \hat{C}_2[\mathrm{SO}(6)] + \kappa_4 \hat{C}_2[\mathrm{SO}(5)] + \kappa_5 \hat{C}_2[\mathrm{SO}(3)]\end{aligned}$$

The coefficients η and κ are certain combinations of the coefficients ε and υ.

The solvable IBM Hamiltonians

Excitation spectrum of H_{IBM} is determined by

$$\hat{H}_{\text{IBM}} = \eta_1 \hat{C}_1[\text{U}(5)] + \kappa_1 \hat{C}_2[\text{U}(5)] + \kappa_2 \hat{C}_2[\text{SU}(3)]$$
$$+ \kappa_3 \hat{C}_2[\text{SO}(6)] + \kappa_4 \hat{C}_2[\text{SO}(5)] + \kappa_5 \hat{C}_2[\text{SO}(3)]$$

If certain coefficients are zero, H_{IBM} can be written as a sum of commuting operators:

$$\hat{H}_{\text{U}(5)} = \eta_1 \hat{C}_1[\text{U}(5)] + \kappa_1 \hat{C}_2[\text{U}(5)] + \kappa_4 \hat{C}_2[\text{SO}(5)] + \kappa_5 \hat{C}_2[\text{SO}(3)]$$
$$\hat{H}_{\text{SU}(3)} = \kappa_2 \hat{C}_2[\text{SU}(3)] + \kappa_5 \hat{C}_2[\text{SO}(3)]$$
$$\hat{H}_{\text{SO}(6)} = \kappa_3 \hat{C}_2[\text{SO}(6)] + \kappa_4 \hat{C}_2[\text{SO}(5)] + \kappa_5 \hat{C}_2[\text{SO}(3)]$$

The U(5) vibrational limit

U(5) Hamiltonian:

$$\hat{H}_{\mathrm{U}(5)} = \varepsilon \hat{n}_d + \sum_{L=0,2,4} c_L \tfrac{1}{2}\left(d^+ \times d^+\right)^{(L)} \cdot \left(\tilde{d} \times \tilde{d}\right)^{(L)}$$

Energy eigenvalues:

$$E(n_d, \upsilon, L) = \varepsilon n_d + \kappa_1 n_d (n_d + 4) + \kappa_4 \upsilon(\upsilon + 3) + \kappa_5 L(L+1)$$

with

$$\kappa_1 = \tfrac{1}{12} c_0$$

$$\kappa_4 = -\tfrac{1}{10} c_0 + \tfrac{1}{7} c_2 - \tfrac{3}{70} c_4$$

$$\kappa_5 = -\tfrac{1}{14} c_2 + \tfrac{1}{14} c_4$$

The features of *SU(5)* limit:

[Iachello and Arima, (1987) and Iachello and Arima (1976)]

a) The triplet of 0_2^+, 2_2^+, 4_1^+ states in neighbourhood of ***twice*** the energy of the first excited 2^+ state ($=E2_g^+$).

b) The quadrupole moment of the first excited state, denoted as $\boldsymbol{Q(2_g^+)= 0}$.

c) The energy ratio $\mathbf{R_{42} (= E4_g^+ /E2_g^+)= 2.0}$

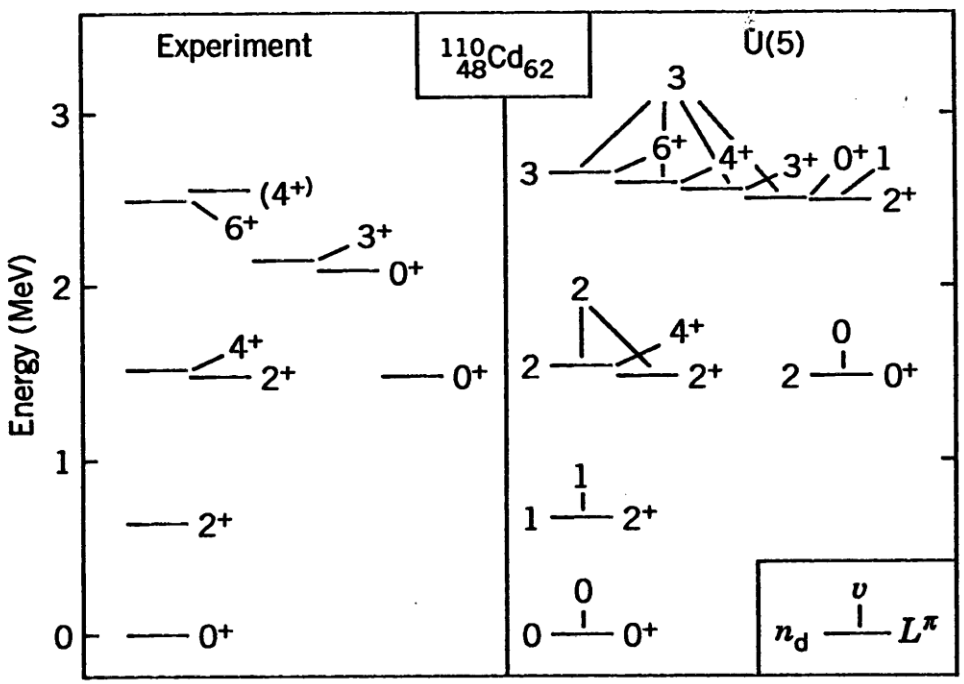
d) The γ-band exhibits a staggering pattern in its energy levels with states like $2\gamma+$, ($3\gamma+$, $4\gamma+$), ($5\gamma+$, $6\gamma+$), and so on. In contrast, ***the tri-axial rotor*** with an asymmetry parameter (γ_0) displays a different staggering pattern with states like ($2\gamma+$, $3\gamma+$), ($4\gamma+$, $5\gamma+$), and so on.

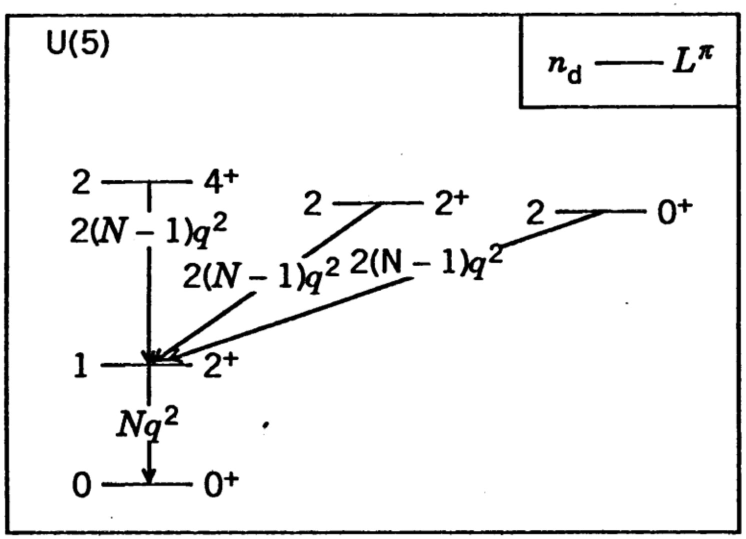
e) Two nuclei that serve as ***excellent examples*** of *SU(5)* type nuclei are ^{64}Zn and ^{76}Se.

The U(5) vibrational limit

Anharmonic vibration spectrum associated with the quadrupole oscillations of a spherical surface.

Conserved quantum numbers: n_d, υ, L.





A. Arima & F. Iachello, Ann. Phys. (NY) **99** (1976) 253
D. Brink *et al.*, Phys. Lett. **19** (1965) 413

The SU(3) rotational limit

SU(3) Hamiltonian:

$$\hat{H}_{\mathrm{SU(3)}} = a\hat{Q}_{\chi} \cdot \hat{Q}_{\chi} + b\hat{L} \cdot \hat{L}$$

Energy eigenvalues:

$$E(\lambda,\mu,L) = \kappa_2\left(\lambda^2 + \mu^2 + 3\lambda + 3\mu + \lambda\mu\right) + \kappa_5 L(L+1)$$

with

$$\kappa_2 = \tfrac{1}{2}a$$

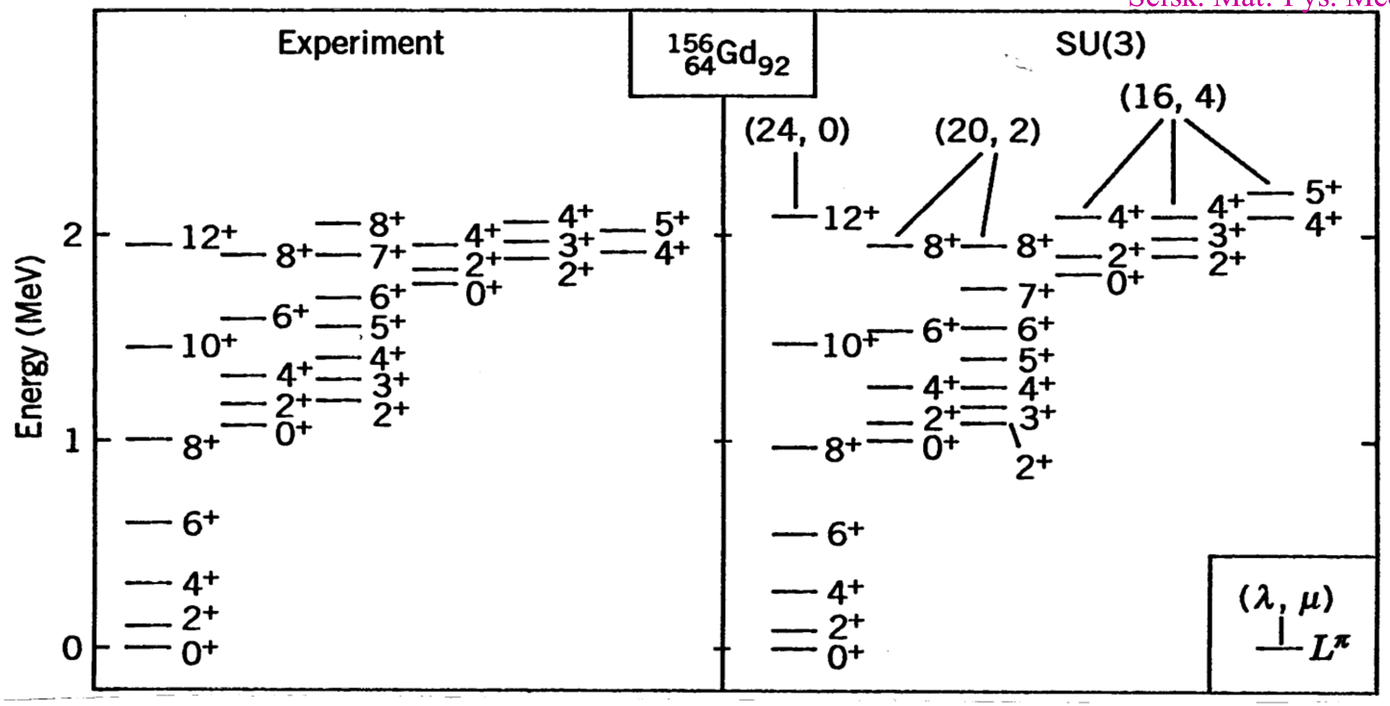
$$\kappa_5 = b - \tfrac{3}{8}a$$

The SU(3) rotational limit

Rotation-vibration spectrum of quadrupole oscillations of a spheroidal surface.

Conserved quantum numbers: (λ,μ), L.

A. Arima & F. Iachello, Ann. Phys. (NY) **111** (1978) 201
A. Bohr & B.R. Mottelson, Dan. Vid. Selsk. Mat.-Fys. Medd. **27** (1953) No 16



The features of the SU(3) limit:

[Iachello & Arima (1987) and (1978)]

A) The energy ratio $\boldsymbol{R_{42} = 10/3}$.
B) The energy gap between the two neighbouring levels of the ground state rotational band and the β-vibrational band remains consistent.
C) The energy levels of states with identical spin values (I) are equivalent for both the γ-vibrational and β-vibrational bands. This means that these γ- and β-bands exhibit degeneracy for the same spin states.
D) There exist many bands with different properties: the ground state rotational (g-)band with $K^{\pi} = 0_1^+$, the β-vibrational band with $K^{\pi} = 0_2^+$, the γ-vibrational band with $K^{\pi} = 2_1^+$, the $\beta 2$-vibrational band with $K^{\pi} = 0_3^+$, the $\beta\gamma$-vibrational band with $K^{\pi} = 2_2^+$, $^+$, the $\gamma\gamma$-vibrational band with $K^{\pi} = 4_1^+$, etc.
E) The *band mixing parameter* Z_γ, which relates to the B(E2) values between the γ-band and the g-band, has a value of zero.
F) The ratio of $\boldsymbol{B(E2;\, 0_\beta^+ \rightarrow 2_g^+)\, /\, B\, (E2;\, 2_\gamma^+ \rightarrow 0_g^+) = 1/6}$.
G) The ratio of $\boldsymbol{B(E2;\, 2_\gamma^+ \rightarrow 0_g^+)\, /\, B\, (E2;\, 2_g^+ \rightarrow 0_g^+) = 0}$.

The SO(6) γ-unstable limit

SO(6) Hamiltonian:

$$\hat{H}_{\mathrm{SO(6)}} = a\hat{P}^{+} \cdot \hat{P} + b\hat{T}_3 \cdot \hat{T}_3 + c\hat{L} \cdot \hat{L}$$

Energy eigenvalues:

$$E(\sigma,\upsilon,L) = \kappa_3\left[N(N+4) - \sigma(\sigma+4)\right] + \kappa_4\upsilon(\upsilon+3) + \kappa_5 L(L+1)$$

with

$$\kappa_3 = \tfrac{1}{4}a$$

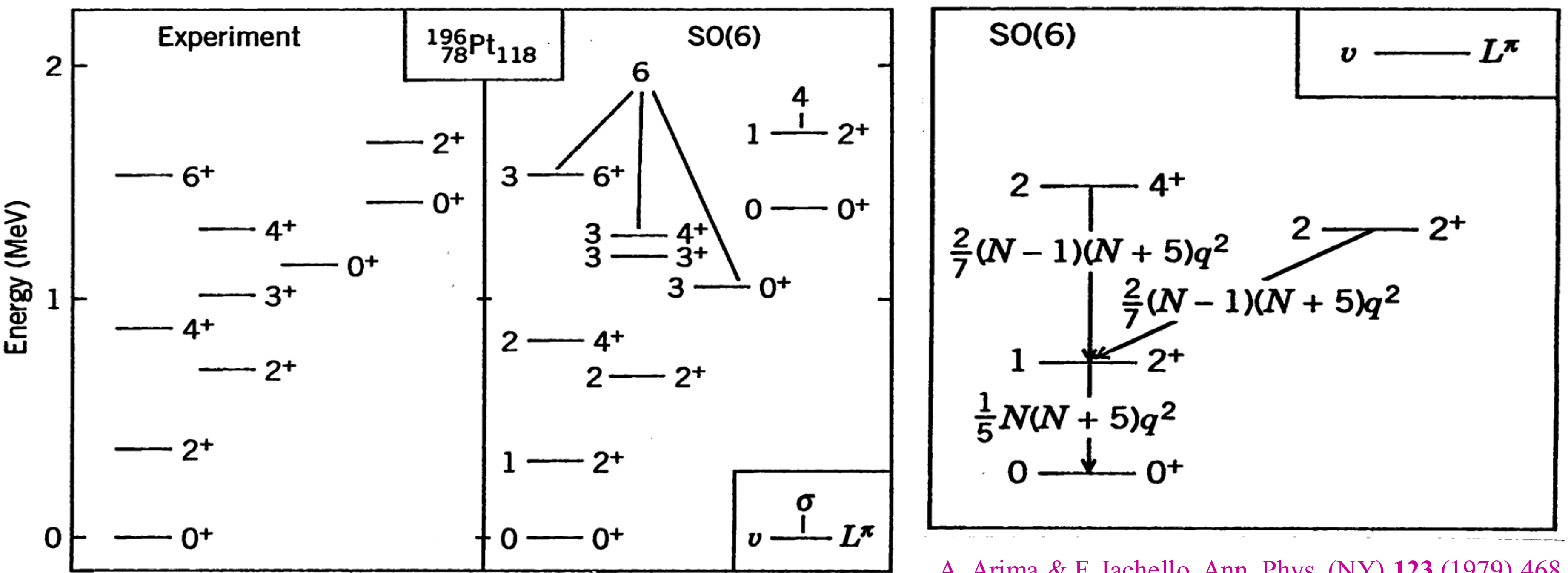
$$\kappa_4 = \tfrac{1}{2}b$$

$$\kappa_5 = -\tfrac{1}{10}b + c$$

The SO(6) γ-unstable limit

Rotation-vibration spectrum of quadrupole oscillations of a γ-unstable spheroidal surface.

Conserved quantum numbers: σ, υ, L.



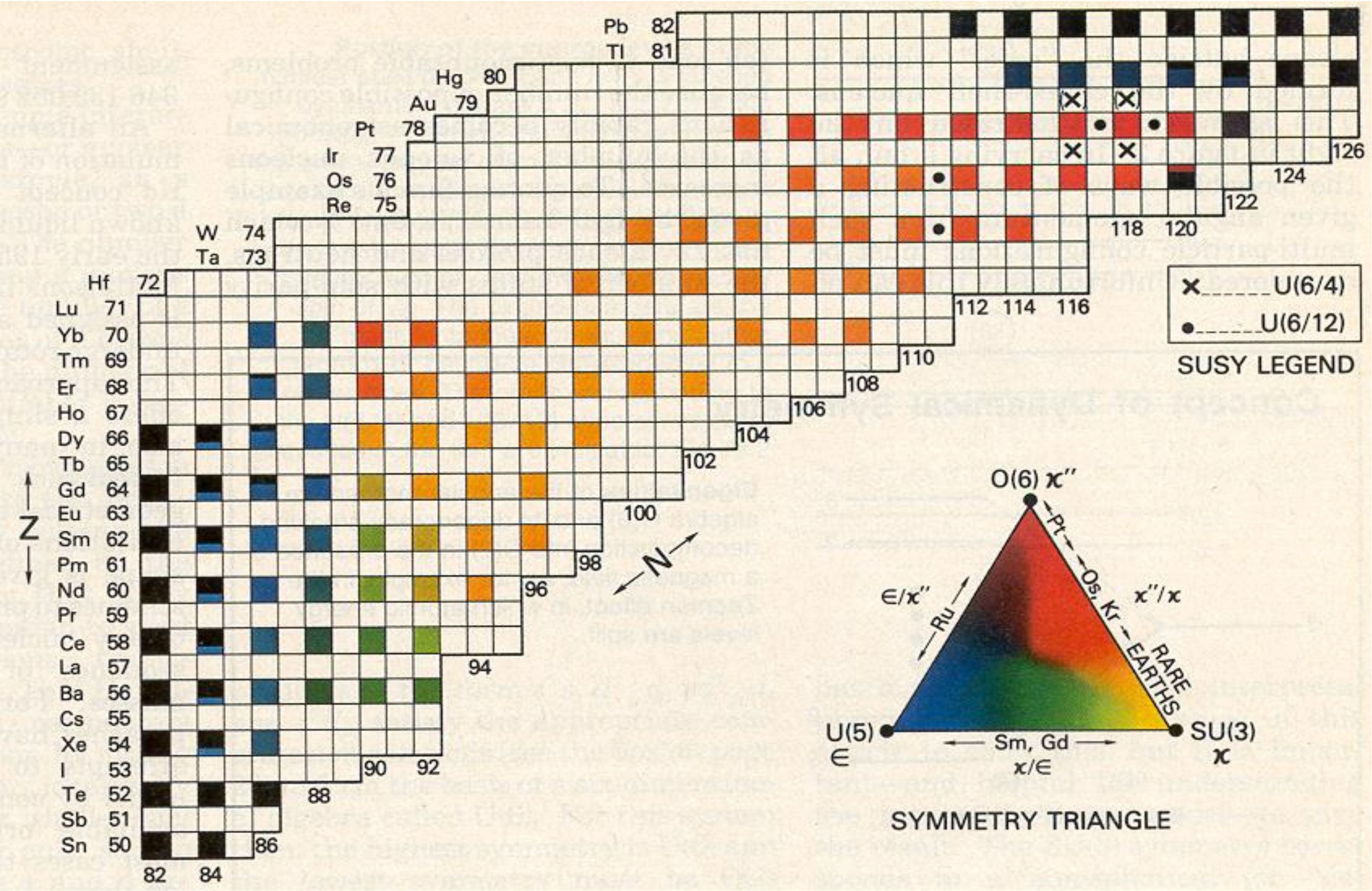
A. Arima & F. Iachello, Ann. Phys. (NY) **123** (1979) 468
L. Wilets & M. Jean, Phys. Rev. **102** (1956) 788

In the *O(6)* limit, the nuclei exhibit the following characteristics:

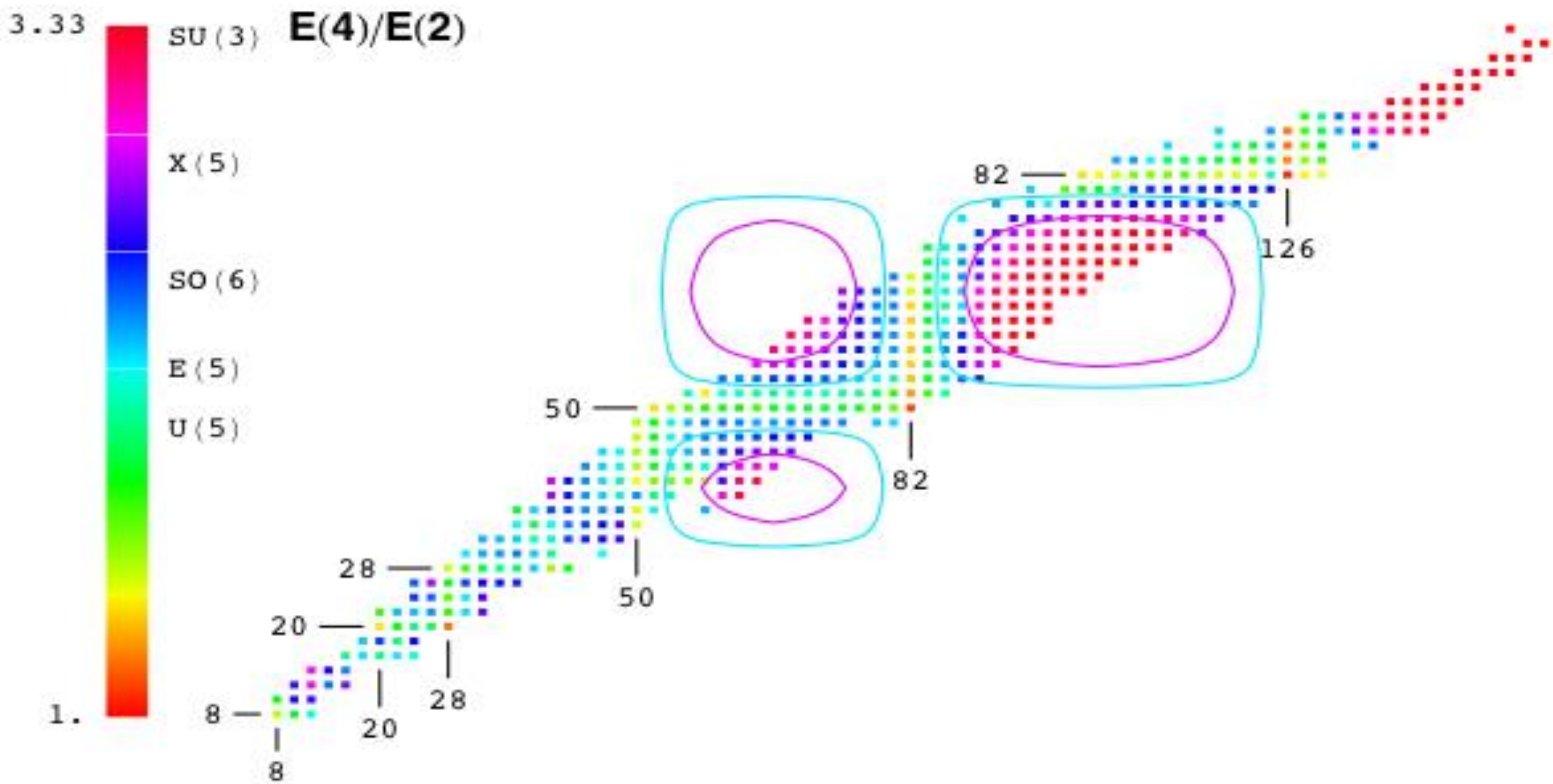
A)The ground state (***g-***) band is denoted as $|N, \sigma = N, \tau, L = 2\tau >$.
B)The ***γ-*** band exhibits an energy level pattern that shows staggering, with states like *2γ+, (3γ+, 4γ+), (5γ+, 6γ+),* and so on. In contrast, the ***tri-axial rotor*** with the asymmetry parameter (γ_0) displays a different staggering pattern, with states like *(2γ+, 3γ+), (4γ+, 5γ+),* and so on.
C)The ***β-*** band follows a sequence of $0^+(\tau=3)$ - $2^+(\tau=4)$ - $2^+(\tau=5)$ with significant energy gaps between these states.
D)The $\boldsymbol{0_\beta^+}$ state is positioned at a lower energy level than the ***3γ+*** state.
E)The operator E(2), represented by $Q_2(\chi = 0)$, follows a selection rule: $\Delta\sigma = 0$, $\Delta\tau = \pm 1$. Consequently, the 0_β^+ state tends to decay primarily to the 2_2^+ state.
F) The ^{134}***Ba*** and ^{196}***Pt*** are the most notable instances of nuclei that adhere to the ***O(6)*** limiting type characteristics.
G)The energy ratio $\boldsymbol{R_{42} = 2.5}$.

[Iachello & Arima (1987) and (1979)]

Applications of IBM

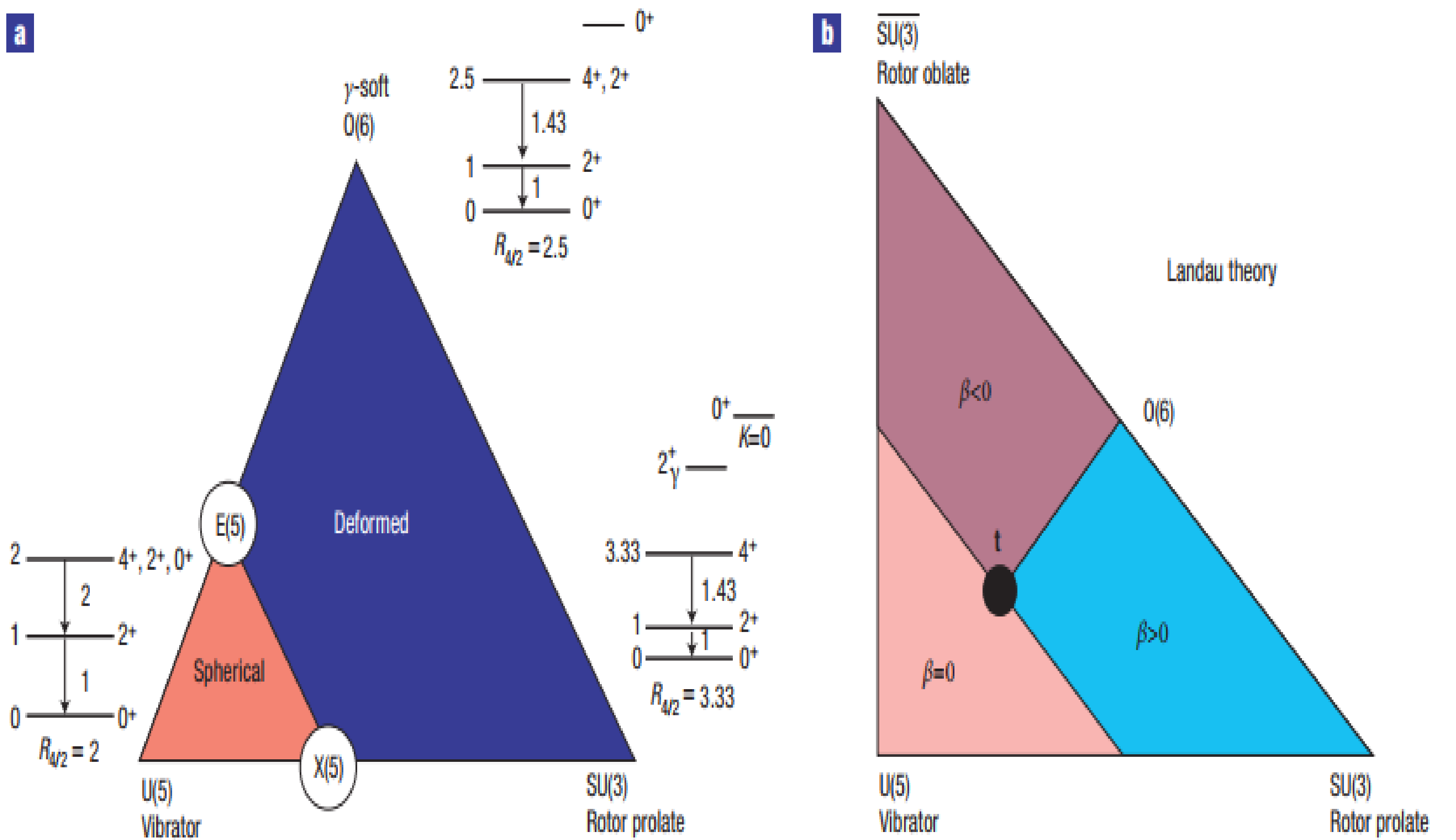


The Ratio R_{42}



In even Z even N nuclei, the energy ratio R_{42} (=E_{4g}^{+}/E_{2g}^{+}) is good measure of deformation and it helps in categorizing the atomic nuclei. For vibrational or SU(5), E(5) symmetry, γ-soft nuclei or SO(6), X(5) symmetry and rotational or SU(3) type nuclei the value of R_4 is 2.0, 2.2, 2.5, 2.9 and 3.33, respectively.

Casten (2006)



a
γ-soft
O(6)
0+
2.5
4+, 2+
1.43
1
2+
1
0
0+
$R_{4/2}$ = 2.5
E(5)
Deformed
2
4+, 2+, 0+
2
1
2+
1
0
0+
$R_{4/2}$ = 2
Spherical
X(5)
U(5)
Vibrator
SU(3)
Rotor prolate
0+
K=0
2^+_γ
3.33
4+
1.43
1
2+
1
0
0+
$R_{4/2}$ = 3.33
b
$\overline{SU(3)}$
Rotor oblate
Landau theory
β<0
O(6)
t
β>0
β=0
U(5)
Vibrator
SU(3)
Rotor prolate

SYMMETRY TRIANGLE FOR NUCLEAR STRUCTURE.

(a) Showing the traditional paradigms at the vertices (along with mini-level schemes), and the two critical point symmetries, E(5) and X(5), at the termini of the phase-transitional region between spherical and deformed nuclei. Note that there are two systems for labelling these paradigms: the geometric language of vibrator, rotor, γ-soft, E(5), and X(5), which are solutions to the Bohr Hamiltonian, and symmetry-based labels from the IBA (U(5), SU(3) and O(6)). This distinction should be borne in mind and is the reason, for example, that E(5) and X(5) are shown in open circles, to distinguish them from the dynamical symmetries at the vertices. Also, even solutions such as U(5) and the vibrator, which appear in both algebraic and geometric approaches, although similar, are not identical.

(b) Extended triangle incorporating oblate shapes interpreted according to Landau theory. **t** represents a nuclear triple point.

The Interacting Boson Model and Calculations

The two body effective Hamiltonian for a system of s- and d- bosons can be written as Eq.1:

$$H = E_0 + \varepsilon'_s (s^\dagger . \tilde{s}) + \varepsilon'_s (d^\dagger . \tilde{d}) + \sum_{L=0,2,4} (1/2)\sqrt{(2L+1)}\, c_L [(d^\dagger x\, d^\dagger)^{(L)} (\tilde{d}\, x \tilde{d})^{(L)}]_0^{(0)} + \frac{1}{\sqrt{2}} \tilde{v}_2 [(d^\dagger x d^\dagger)^{(2)} x (\tilde{d}\, x \tilde{s})^{(2)} + (d^\dagger x s^\dagger)^{(2)}\, x\, (\tilde{d} x \tilde{d})^{(2)}]_0^{(0)}$$

$$+ \frac{1}{2} \tilde{v}_0 [(d^\dagger x d^\dagger)^{(0)} (\tilde{s} x \tilde{s})^{(0)} + (s^\dagger x s^\dagger)^{(0)} (\tilde{d}\, x \tilde{d})^{(0)}]_0^{(0)} + u_2 [(d^\dagger x s^\dagger)^{(2)} x (\tilde{d}\, x \tilde{s})^{(2)}]_0^{(0)} + \frac{1}{2} u_0 [(s^\dagger x s^\dagger)^{(0)} x (\tilde{s}\, x \tilde{s})^{(0)}]_0^{(0)}$$

In above Eq. (1), E_0 is the core energy; ε'_s and ε'_d are the binding energies of s and d bosons energies; c_L, $\tilde{v}_L$ and u_L describe the two-boson interaction. Another form of Hamiltonian is:

$$H' = \varepsilon'' n_d + a_0 (P^\dagger . P) + a_1 (L.L) + a_2 (Q.Q) + a_3 (T_3.T_3) + a_4 (T_4.T_4) \qquad (2)$$

where,

$$n_d = (d^\dagger . \tilde{d}) \qquad P = \frac{1}{2}\{(\tilde{d}\tilde{d}) - (\tilde{s}\tilde{s})\} \qquad L = \sqrt{10}\, \left(d^\dagger x\, \tilde{d}\right)^{(1)} \qquad Q = [(d^\dagger x\, \tilde{s}) + (s^\dagger x\, \tilde{d})]^{(2)} - \frac{1}{2}\sqrt{7}[d^\dagger x \tilde{d}]^{(2)}$$

$$T_3 = [(d^\dagger . \tilde{d})]^{(3)} \qquad T_4 = [(d^\dagger x\, \tilde{d})]^{(4)}$$

A least square fitting technique is used to find out the optimized values of the four parameters i.e., *ε''*, a_0, a_1 and a_2; while $a_3 = a_4 = 0$, for a nucleus lies on *SU(5)* to *SU(3)* transition in Eq. (2). The PHINT programme (Scholten, 1979a) is used to fit the observed energy spectra of a nucleus. All levels with reliable spin assignment ($I^\pi \leq 10^+$) are to be included up to the point that the first level with an uncertain spin assignment appears. In fitting of the energy spectra, we first determine the four parameters of *H'* as discussed above, that reproduce the best lower and higher bands.

The optimized values of these four boson- boson interaction parameters with *E2SD* $(= \alpha_2)$ and *E2DD* $(= \sqrt{5}\beta_2)$ are the input for the FBEM programme (Scholten, 1979b). The E2 transition operator depends upon two parameters α_2 and β_2 as given below:

$$T(E2) = \alpha_2 [d^\dagger + s^\dagger]^{(2)} + \beta_2[d^\dagger]^{(2)}$$

where, α_2 is called the boson effective charge, simply the scaling parameter and affecting the B(E2) values and β_2 accounts for nuclear shape transition. The ratio E2DD/ E2SD =2.958 in the SU(3) limit and reduced to zero in the O(6) limit. The FBEM program (Scholten, 1979b) gives the B(E2) values and ratios.

A. Arima and F. Iachello , *Advances in Nuclear Physics*, edited by J. W. Negela and E. Vogts (Plenum Press, New York), Vol. 13, 1984.

F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge), 1987.

R. F. Casten, *Nuclear Structure from a Simple Perspective*, (Oxford University Press, New York) 1990.

O. Scholten, Programme PHINT, *KVI internal report* **63** (1979a).

O. Scholten, Programme FBEM, *KVI internal report* **63** (1979b).

Old References:

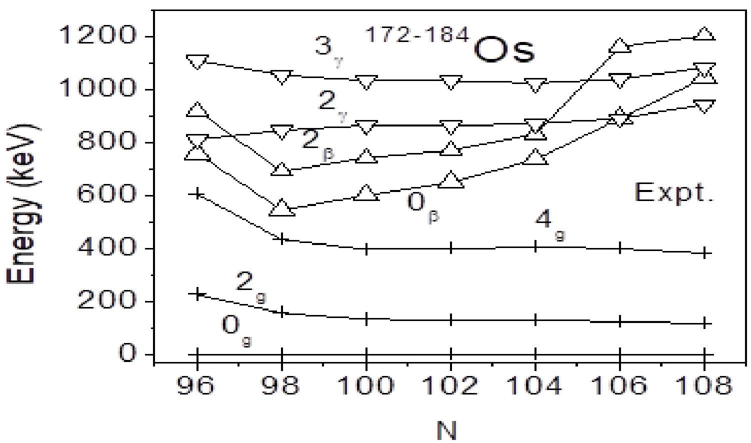
Sr. No	Nuclear Observable	Nucleus	Model	Authors	Reference
1	E (g-, β-, γ- bands) B(E2) ratios	^{102}Ru, ^{110}Cd, ^{188}Pt, ^{110}Pd $^{124\text{-}132}$ Xe	IBM-1 SU(5)	Arima and Iachello	Annals of Physics 99 (1976) 253
2	E (g-, β-, γ- bands)	^{156}Gd, ^{170}Er, ^{234}U	IBM-1 SU(3)	Arima and Iachello	Annals of Physics 111 (1978) 201
3	E (g-, β-, γ- bands) B(E2; 21 →01), B(E2; 41 →21), B(E2; 22 →01), B(E2; 23 →01), $Q2_1$ B(E2; 41 →21)/ B(E2; 21 →01), B(E2; 22 →01/21), B(E2; 22 →41/21), B(E2; 23 →01/21), B(E2; 22 →21/41), B(E2; 31 →21/41), Isomer shifts $\delta\langle r^2 \rangle$ E0 matrix elements, E2/M1 mixing ratios g-factors ($g2_1$) B(E4), etc	$^{146\text{-}156}$Sm	IBM-1 SU(5) to SU(3)	Scholten and Iachello	Annals of Physics 115 (1978) 325
4	E (g-, β-, γ- bands) B(E2) values	^{132}Ba, ^{196}Pt ^{194}Pt	IBM-1 O(6)	Arima and Iachello	Annals of Physics 123 (1979) 468
5	E2/M1 ratios	$^{146\text{-}152}$Sm, $^{152\text{-}156}$Gd, ^{162}Dy, $^{162\text{-}168}$Er, ^{172}Yb, ^{190}Os	IBM-1	Lipas et al.,	NPA 469 (1987) 348

New References:

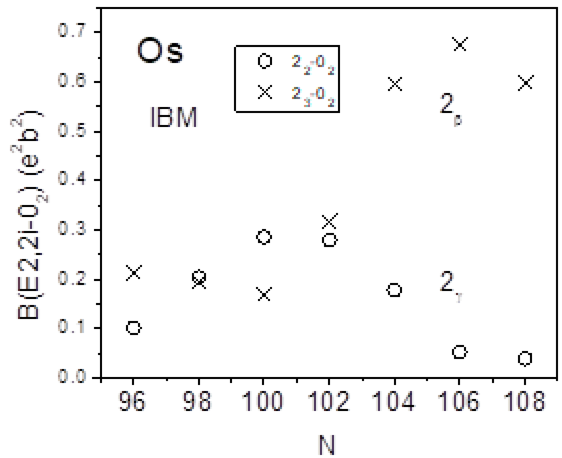
6	B (E2; 01 →21), Isomer shifts δ<r2> = <r2>21 - <r2>01	$^{148\text{-}154}$Sm $^{152\text{-}160}$Gd	Mesoscopic Systems, IBM	Iachello and Zamfir	PRL 92(21) (2004) 212501-1
7	E (g-, β-, γ- bands) B(E2) values, Q	^{100}Mo	IBM-1	Ghafoor and Shawn	JUBPAS 31(2) (2023) 176
8	E (g-, β-, γ- bands) B(E2) values, Q	$^{164-184}$Os	IBM-1	Gupta-Katoch-Sharma	NPA 1041 (2024) 122765
9	E (g-, β-, γ- bands) B(E2) values, Q	W (N=86–118)	IBM-1	Gupta-Katoch-	NPA 1057 (2025) 123032

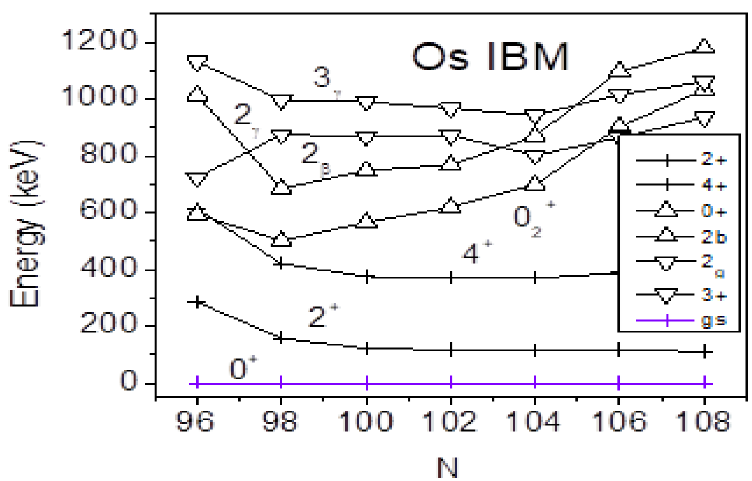
Partial Energy level spectrum of N = 96-108Os in EXPT and IBM-1.

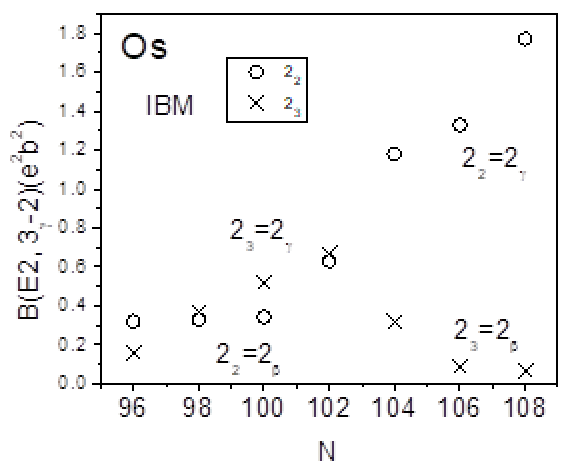
[Gupta- Katoch -Sharma, NPA 1041 (2024) 122765]



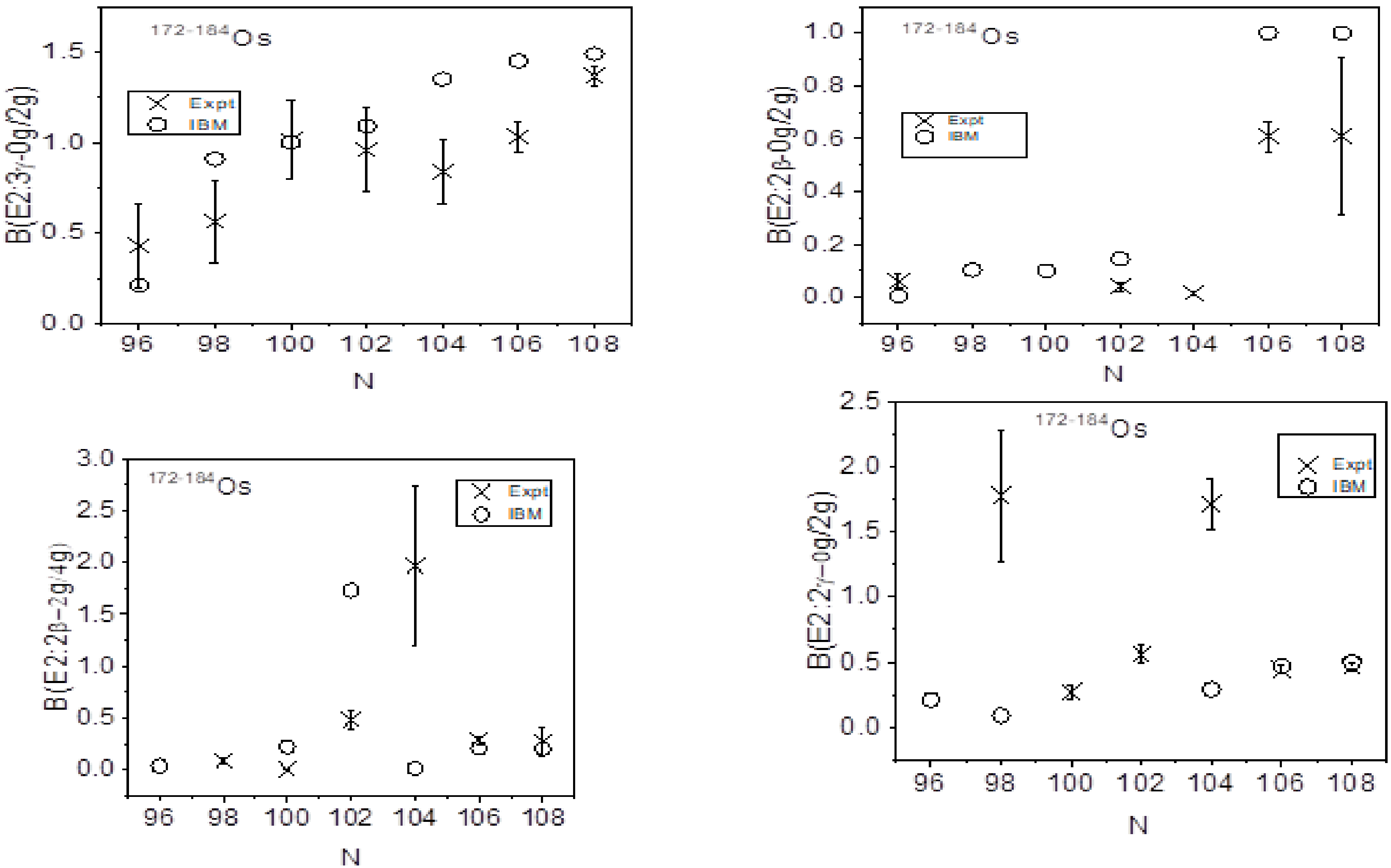
The B(E2) Values in IBM



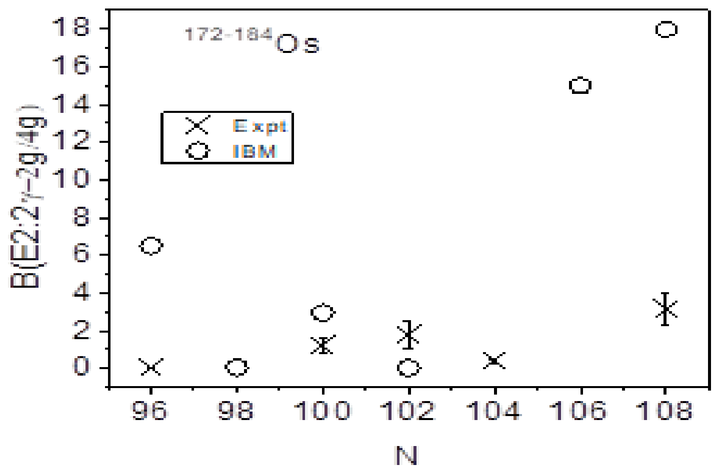




Variation of B(E2) ratios for Os
[Gupta- Katoch -Sharma, NPA 1041 (2024) 122765]



The B(E2; 2γ→2g/4g) vs. N. Fitted parameters (in keV). EPS=Ɛ, QQ=2k, ELL=k', PAIR=k"/2.



N	EPS	QQ	ELL	PAIR
96	479.1	-16.4	1.7	2.7
98	458	-19.7	8.1	8.0
100	443	-20.0	11.2	8.6
102	436	-18.0	14.1	10.3
104	368	-16.5	20.5	10.0
106	353	-18.7	27.8	20.5
108	321.4	-22.7	19.8	20.2

The MULT form of the IBM-1 Hamiltonian consists of 4 terms :

HIBM = *Ɛ nd* + *k Q.Q* + *k'L.L* + *k" P.P*

In PHINT package, *Ɛ = EPS, 2k = QQ, k' = ELL, k"/2 = PAIR* .

The IBM-1 is used to reproduce the β- and γ-bands. The IBM-1 parameters used are given. The inter band B(E2) ratios in IBM-1 are compared with experiment Thus our presentation gives a detailed description of these Os isotopes, and high light their special spectral features. These spectra are similar to those of W and Hf nuclei in quadrant-2. The *N*=96-108 *Os* isotopes display the rich structure of ground-, *β*- and *γ*- bands. The *N*=100 ^{176}Os is *X*(5) nucleus, beyond which a saturation of deformation is observed. The *β*-band is low and lies below the *γ*- band. All these *Os* isotopes are prolate shaped.

Thank you.